Travelling waves solution of the unsteady problem of binary gas mixture affected by a nonlinear thermal radiation field

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Abstract: In the present study, a development of the paper [Can. J. of Phy., 2012, 90(2): 137-149] is introduced. The non-stationary BGK (Bhatnager- Gross- Krook) model of the Boltzmann nonlinear partial differential equations for a rarefied gas mixture affected by nonlinear thermal radiation field, for the first time, are solved instead of the stationary equations. The travelling wave solution method is used to get the exact solution of the nonlinear partial differential equations. These equations were produced from applying the moment method to the unsteady Boltzmann equation. Now, nonlinear partial differential equations should be solved in place of nonlinear ordinary differential equations, which represent an arduous task. The unsteady solution gives the problem a great generality and more applications. The new problem is investigated to follow the behavior of the macroscopic properties of the gas mixture such as the temperature and concentration. They are substituted into the corresponding two stream Maxiwallian distribution functions permitting us to investigate the non-equilibrium thermodynamic properties of the system (gas mixture + the heated plate). The entropy, entropy flux, entropy production, thermodynamic forces, kinetic coefficients are obtained for the mixture. The verification of the Boltzmann H-theorem, Le Chatelier principle, the second law of thermodynamic and the celebrated Onsager's reciprocity relation for the system, are investigated. The ratios between the different contributions of the internal energy changes based upon the total derivatives of the extensive parameters are estimated via the Gibbs formula. The results are applied to the Argon-Neon binary gas mixture, for various values of both of the molar fraction parameters and radiation field intensity. Graphics illustrating the calculated variables are drawn to predict their behavior and the results are discussed.

Keywords: Binary Gas Mixture, Radiation Field, Exact Solutions, Travelling Wave Method, Unsteady BGK Model,
Boltzmann Kinetic Equation, Moments Method, Liu-Lees Model, Boltzmann H-Theorem,
Irreversible Thermodynamics

1. Introduction

It should be noted that the renewed interest to internal rarefied flows has been motivated by the recent development in the micro-electro-mechanical systems (MEMS) technology. In most MEMS applications, the continuum equations are applied with the appropriate slip boundary conditions [1, 2]. It is evident; however, that the approaches based on kinetic theory are more suitable to solve such type of the problems, since the whole range of the gas rarefaction can be studied in a uniform manner [3].

In spite of the fact that, the statistical-mechanical study of fluid mixtures far from equilibrium is a very interesting subject from a theoretical as well as a practical viewpoint. There are very few articles in the literature concerning this topic [4-15], compared to the huge amount of papers in the case of a single gas, see e.g. [16-23].

The general description of the system of gas mixture is much more complicated than that for a single fluid case. Not only the number of transport coefficients is much higher, but also they are functions of parameters such as the molar fractions, the mass ratios, and the size ratios. Due to the complexity of the general problem, tractable specific situations must be considered [24]. The Boltzmann equation provides the appropriate framework for analyzing non-equilibrium states in dilute gases. Nevertheless, due to its mathematical intricacy, only a few exact solutions are known [25-27]. The difficulties are much greater when the system is constituted by particles of different species, as one has to deal with a set of coupled unsteady Boltzmann

equations [28]. Furthermore, the transport properties depend on the molar fractions, the mass ratios, and the size ratios, in order to get explicit results [29,30]. As it is known, not the Boltzmann equation itself but some kinetic model is used in solving the majority of practical problems of the kinetic theory of gases [31]. This circumstance is due to the fact that the Boltzmann collision integral has a quite complex structure. One of the most significant achievements in rarefied gas theory in the last century is the Krook model for the Boltzmann equation [32].The Krook model relaxation equation retains all the features of the Boltzmann equation which are associated with free molecular motion and describes approximately, in a mean-statistical fashion, the molecular collisions. The structure of the collision term in the Krook formula is the simplest of all possible structures which reflect the nature of the phenomenon. Careful and thorough study of the model relaxation equation [33], and also solution of several problems for this equation, have aided in providing a deeper understanding of the processes in a rarefied gas.

Since, every material in this universe, and hence also the particles of a gas absorb and radiate thermal energy, the interaction with the thermal radiation that is contained within the volume of the body may be important in gases. Since the latter unlike solids and liquids are capable of undergoing conspicuous volume changes. Taking this interaction into account makes the behavior of the gases more realistic. Radiative heat transfer in gases has important applications from combustion systems to modeling atmospheric processes. The magnitude of radiative heat fluxes can have profound effects on combustion performance and environmental impact. For this purpose, Abourabia and Abdel Wahid [21], and Abdel Wahid [34] have introduced a new approach for studying the influence of a thermal radiation field upon a rarefied neutral gas. This idea was applied to a steady [21] and unsteady problem [34] of the half space filled by a neutral gas specified by a flat rested heated plate in a frame co-moving with the gas. I will extend these papers to study the unsteady problem for a rarefied binary gas mixture affected by nonlinear thermal radiation field, for the best of my knowledge; this study is done for the first time at all.

My aim in this paper is as follows: first, in section (2) to introduce this new unsteady approach for studying the influence of thermal radiation field on a rarefied neutral binary gas mixture. For this purpose, we use a coupled system of unsteady kinetic Boltzmann equations, one for each component of the binary mixture. I insert the radiation field effect into the force term of the Boltzmann equation as a radiation force. I follow this approach, using Liu-Lees model for two stream Maxwellian distribution functions and the moment method [16-18] to predict the behavior of the macroscopic properties of the binary gas mixture and various radiation field intensity due to different plate temperatures; such as the temperature and concentration which are in turn substituted into the corresponding distribution functions. This approach, in section (3), will permit us to study the behavior of the equilibrium and non-equilibrium unsteady distribution functions for various values of the molar fraction parameters.

The important non-equilibrium thermodynamic properties of the system (binary gas mixture + heated plate) are calculated. Particularly, I obtain the entropy, entropy flux, entropy production, thermodynamic forces, kinetic coefficients .I investigate the verification of the second law of thermodynamic, Boltzmann H-theorem, the Onsager's reciprocity relation. The ratios between the different contributions of the internal energy changes based upon the total derivatives of the extensive parameters are predicted via the Gibbs' formula. Section (4) shows the results and the discussion of applying the results to the Argon-Neon binary gas mixture. Finally, Section (5) indicates the important conclusions of the paper.

2. The Physical Problem and Mathematical Formulation

Consider a binary gas mixture consisting of monatomic molecules of two types, say components A and B, fills the upper half of the space ($y \geq 0$), which is bounded by an infinite immobile flat plate (y=0), in a uniform pressure P_S [21,35]. The plate is heated suddenly to produce thermal radiation field. The flow is considered unsteady, and compressible. In a frame co-moving with the fluid the behavior of the binary gas mixture is studied under the assumptions that:

(i)   At the rested plate boundary, the velocities of the incident and reflected particles are equal; but of opposite sign. This is happened according to Maxwell formula of momentum defuse reflection. On the other hand the exchange will be due to only the temperature difference between the particles and the heated plate, taking the form of full energy accommodation [36].

(ii)   The gas is considered gray absorbing-emitting but not a scattering medium.

(iii) A thermal radiation force is acting from the heated plate on the binary atomic gas mixture, written in the vector notation [37-39] as

$$\vec{F} = \frac{-4\sigma_s}{3n_s c}\vec{\nabla}T^4(y,t) \Rightarrow F_y = \frac{-16\sigma_s T^3}{3n_s c}\frac{\partial T(y,t)}{\partial y} \qquad (1)$$

for unsteady motion, the process in the system under study subject to a thermal radiation force F_y can be expressed in terms of the Boltzmann kinetic equations in the BGK model [16,18, 33] written in the unsteady form, for the first time,:

$$\frac{\partial f_A}{\partial t}+C_y\frac{\partial f_A}{\partial y}+\frac{F_y}{m_A}\frac{\partial f_A}{\partial C_y}=\nu_{AA}(f_{0A}-f_A)+\nu_{AB}(f_{0B}-f_A), \quad (2)$$

$$\frac{\partial f_B}{\partial t}+C_y\frac{\partial f_B}{\partial y}+\frac{F_y}{m_B}\frac{\partial f_B}{\partial C_y}=\nu_{BB}(f_{0B}-f_B)+\nu_{BA}(f_{0A}-f_B), \quad (3)$$

where C_y , f_β and m_β are the velocity of the gas particles component along y-axis, the two–stream Maxwellian

distribution function and mass of particles of the β type and V_{AA}, V_{AB}, V_{BB} and V_{BA} are the collision frequencies which are given in [18,40,41] as:

$$V_{\alpha\alpha} = n_{s\alpha}\sigma_{\alpha\alpha}V_{p\alpha}$$

and

$$V_{\alpha\beta} = n_{s\alpha}\sigma_{\alpha\beta}V_{p\alpha\beta}$$

where

$$V_{p\alpha} = \left[\sqrt{\frac{8KT_s}{\pi m_\alpha}}\right] \quad,\quad V_{p\alpha\beta} = \left[\sqrt{\frac{8KT_s}{\pi \mu_{\alpha\beta}}}\right] \quad,\quad \mu_{AB} = \left(\frac{m_A m_B}{m_A+m_B}\right) \quad,$$

$\sigma_{AB} = \pi d_{eff}^{2}, d_{eff} = (\frac{d_A+d_B}{2})^2$, ($\alpha$ and $\beta \equiv A$ or B), where $V_{p\alpha}$, $\mu_{\alpha\beta}$, d_{eff}, $n_{s\alpha}$ and $\sigma_{\alpha\beta}$ are the mean velocity of particles, the reduced mass, the diameter of the effective collisions sphere, gas concentration at the plate surface, and effective collisions cross section for α and β types, respectively and $f_{0\beta}$ are the local Maxwellian distribution functions denoted by

$$f_{0\beta} = \frac{n_\beta}{\left(2\pi RT_\beta\right)^{\frac{3}{2}}} \exp\left[\frac{-C^2}{\left(2RT_\beta\right)}\right],$$

where R is the gas constant and $C^2 = C_x^2 + C_y^{2} + C_z^2$.

Lee's moment method [42-43], for the solution of the Boltzmann equation, is employed. When the application of heat to a gas causes it to expand, it is thereby rendered rarer than the neighboring parts of the gas; and it tends to form an upward current of the heated gas, which is of course accompanied with a current of the more remote parts of the gas in the opposite direction. The fresh portions of gas are brought into the neighborhood of the source of heat, carrying their heat with them into other regions [44]. We assume that the temperature of the upward going gas particles is $T_{1\beta}$, while the temperature of the downward going gas particles is $T_{2\beta}$ [21, 25]. The corresponding concentrations are $n_{1\beta}$ and $n_{2\beta}$. We use the Liu-Lees model of the two–stream Maxwellian distribution function near the plate [36,45] for particles of the β type, which can be represented as:

$$f_\beta = \begin{cases} f_{1\beta} = \dfrac{n_{1\beta}}{\left(2\pi RT_{1\beta}\right)^{\frac{3}{2}}} exp[\dfrac{-C^2}{\left(2RT_{1\beta}\right)}], \text{For } C_y > 0 \quad \uparrow \\[4mm] f_{2\beta} = \dfrac{n_{2\beta}}{\left(2\pi RT_{2\beta}\right)^{\frac{3}{2}}} exp[\dfrac{-C^2}{\left(2RT_{2\beta}\right)}], \text{For } C_y < 0 \quad \downarrow \end{cases} \quad (4)$$

The velocity distribution function f is not of direct interest to us, in this stage, but the moments of the distribution function. Therefore, we derived the Maxwell's moment equations by multiplying the Boltzmann equation by a function of velocity $Q_i(C)$ and integrating over the velocity space. How many and what forms of Q_i will be used is dependent on how many unknown variables need to be determined and how many equations need to be solved.

Multiplying Eqs.(2,3) by some functions of velocity $Q_i = Q_i(C)$, and integrating w. r. t. C, taking into consideration the discontinuity of the distribution function, caused by the cone of influence [36], the resulting equation can then be written as follows; for particles of each gas component β with the second one α type,

$$\frac{\partial}{\partial t}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} Q_i\, f_{2\beta}\, dC + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} Q_i\, f_{1\beta}\, dC\right] +$$
$$\frac{d}{dy}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} Q_i C_y\, f_{2\beta}\, dC + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} Q_i C_y\, f_{1\beta}\, dC\right]$$
$$-\frac{F_y}{m_\beta}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_{2\beta}\,\frac{dQ_i}{dC_y}\, dC + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_{1\beta}\,\frac{dQ_i}{dC_y}\, dC\right] = \quad (5)$$
$$V_{\beta\beta}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q_i f_{0\beta}\, dC - \left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_{2\beta} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_{1\beta}\right)Q_i\, dC\right] +$$
$$V_{\beta\alpha}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q_i f_{0\alpha}\, dC - \left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_{2\beta} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_{1\beta}\right)Q_i\, dC\right],$$

where $dC = dC_x dC_y dC_z$.

Equations (5) are called the general equations of transfer [46]. We obtain the dimensionless forms of the variables by taking:

$$y = \bar{y}\left(\tau_{AB}\sqrt{2RT_s}\right), t = \bar{t}\,\tau_{AB}, \vec{C} = \bar{\vec{C}}\sqrt{2RT_s},$$
$$f_{i\beta} = \frac{\bar{f}_{i\beta}(2\pi RT_s)^{\frac{3}{2}}}{n_s}, i = 0,1,2, N_\beta = \frac{16\sigma_s T_s^3}{3n_s cm_\beta R}, \quad (6)$$
$$T_{1\beta} = \bar{T}_{1\beta}T_s \quad, \quad n_{1\beta} = \bar{n}_{1\beta}n_s \quad, \quad T_{2\beta} = \bar{T}_{2\beta}T_s \quad,$$
$$n_{2\beta} = \bar{n}_{2\beta}n_s, \text{and} \quad dU_\beta = \overline{dU}_\beta K_b T_s.$$

where τ_{AB}, N_β and dU_β are the relaxation time between collisions of the A-B species, non-dimensional constant and internal energy change of the gas species of the β type, respectively.

It is assumed that the temperature differences within the gas are sufficiently small such that $T^4(y,t)$ may be expressed as a linear function of the temperature. This is accomplished by expanding F_y in a Taylor series about T_∞ and neglecting higher-order terms [8-9,32-33], thus $T^4(y,t) \cong 4T_\infty^3 T(y,t) - 4T_\infty^4$. This implies that:

$$F_y = \frac{-4\sigma_s}{3n_s c}\frac{\partial}{\partial y}\left[4T_\infty^3 T(y,t) - 4T_\infty^4\right] = \frac{-16\sigma_s T_\infty^3}{3n_s c}\frac{\partial T(y,t)}{\partial y}$$

Once the expressions for $f_{0\beta}, f_{1\beta}$ and $f_{2\beta}$ are introduced, macroscopic quantities such as density, velocity, temperature, etc… can be computed from the appropriate weighted integral of the distribution functions as follows[36,47] ;

Number density:

$$n_\beta(y,t)=\int f_\beta(y,t,C_y)d\underline{C}=\frac{(n_{1\beta}+n_{2\beta})}{2} \qquad (7)$$

Hydrodynamic (bulk) velocity:

$$u_\beta(y,t)=\frac{1}{n_\beta}\int C_y f_\beta(y,t,C_y)d\underline{C}=$$
$$\frac{(n_{1\beta}\sqrt{T_{1\beta}}-n_{2\beta}\sqrt{T_{2\beta}})}{(n_{1\beta}+n_{2\beta})} \qquad (8)$$

Temperature:

$$T_\beta(y,t)=\frac{1}{3n_\beta}\int C^2 f_\beta(y,t,C_y)d\underline{C}=$$
$$\left(\frac{n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}}{n_{1\beta}+n_{2\beta}}\right) \qquad (9)$$

The static pressure normal to the plate:

$$P_{yy\beta}(y,t)=\int C_y^{\,2}\,f_\beta(y,t,C_y)d\underline{C}=$$
$$\frac{1}{2}(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}) \qquad (10)$$

The heat flux component:

$$Q_{y\beta}(y,t)=\int C_y C^2 f_\beta(y,t,C_y)d\underline{C}=$$
$$\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right) \qquad (11)$$

In Eq.(4) there are four unknown functions, $T_{1\beta}(y,t),T_{2\beta}(y,t),n_{1\beta}(y,t)$ and $n_{2\beta}(y,t)$, needed to be determined for each component in the binary gas mixture. Thus, we need four equations for each one, the equation of state, two moment equations, and with the condition that, we shall study the problem in a coordinate system of the phase space in which the bulk velocity u is located at the origin, to conform a complete set to solve the problem. Taking $Q_i=C^2$ and $\frac{1}{2}C_y C^2$, and substituting formula (4) into the transfer Eq.(5), taking (6) into consideration, to get the moments of the equations.

We note that for a neutral gas, this procedure will give rise to $Q_1=C^2$ (energy conservation) and $Q_2=\frac{1}{2}C_y C^2$ (energy transport). After dropping the bars we get eight equations, four for each component of the binary gas mixture as follows: For particles of the $\beta=A,B$ types, the conservation of energy

$$\frac{\partial}{\partial t}\left(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}\right)+\frac{\partial}{\partial y}\left[\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)\right]$$
$$+N_\beta\left(n_{1\beta}T_{1\beta}^{\frac{1}{2}}-n_{2\beta}T_{2\beta}^{\frac{1}{2}}\right)\frac{\partial}{\partial y}\left(\frac{n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}}{n_{1\beta}+n_{2\beta}}\right)=0, \qquad (12)$$

and the heat flux component in the y-direction

$$\frac{\partial}{\partial t}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)+\frac{5}{4}\frac{\partial}{\partial y}\left[\left(n_{1\beta}T_{1\beta}^{2}+n_{2\beta}T_{2\beta}^{2}\right)\right]+$$
$$\frac{3}{2}N_\beta\left(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}\right)\frac{\partial}{\partial y}\left(\frac{n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}}{n_{1\beta}+n_{2\beta}}\right)=$$
$$\left[\frac{\nu_{\beta\beta}}{\nu_{AB}}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)+\frac{\nu_{\beta\alpha}}{\nu_{AB}}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)\right]. \qquad (13)$$

The above three equations, for both group of equations, corresponding to each species, are complemented by the equations of state [48-50],

$$P_\beta=n_\beta T_\beta=const. \qquad (14)$$

and with the condition that, we shall study the problem in a coordinate system of the phase space in which the bulk velocity u is located at the origin. Thus, using Eq.(8), we get the fourth equation:

$$\left(n_{1\beta}T_{1\beta}^{\frac{1}{2}}-n_{2\beta}T_{2\beta}^{\frac{1}{2}}\right)=0 \qquad (15)$$

I will use the traveling wave solution method [51-52], to solve the problem, considering

$$\xi=ly-mt \qquad (16)$$

Such that to make all the dependent variables as functions of ξ . Here l and m are transformation constants, which do not depend on the properties of the fluid but as parameters to be determined by the boundary and initial conditions [29]. From Eq.(16) we get the derivatives:

$$\frac{\partial}{\partial t}=-m\frac{\partial}{\partial\xi}\,,\,\frac{\partial}{\partial y}=l\frac{\partial}{\partial\xi}\text{ and }\frac{\partial^a}{\partial t^a}=$$
$$(-1)^a m^a\frac{\partial^a}{\partial\xi^a}\,,\,\frac{\partial^a}{\partial y^a}=l^a\frac{\partial^a}{\partial\xi^a} \qquad (17)$$

where a is a positive integer.

Substituting from Eqs. (16-17) into Eqs. (12,13) we get:

$$-m\frac{\partial}{\partial\xi}\left(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}\right)+l\frac{\partial}{\partial\xi}\left[\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)\right]$$
$$+lN_\beta\left(\frac{\partial}{\partial\xi}\left(\frac{n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}}{n_{1\beta}+n_{2\beta}}\right)\right)\left(n_{1\beta}T_{1\beta}^{\frac{1}{2}}-n_{2\beta}T_{2\beta}^{\frac{1}{2}}\right)=0 \qquad (18)$$

and

$$-m\frac{\partial}{\partial\xi}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)+\frac{5}{4}l\frac{\partial}{\partial\xi}\left[\left(n_{1\beta}T_{1\beta}^{2}+n_{2\beta}T_{2\beta}^{2}\right)\right]$$

$$+\frac{3}{2}lN_{\beta}\left(\frac{\partial}{\partial\xi}\left(\frac{n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}}{n_{1\beta}+n_{2\beta}}\right)\right)\left(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}\right)=$$

$$\left[\frac{v_{\beta\beta}}{v_{AB}}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)+\frac{v_{\beta\alpha}}{v_{AB}}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)\right], \tag{19}$$

Know, I intend to solve Eqs.(14,15,18,19) to obtain the solution of the initial and boundary value problem for each of the gas components to estimate the four unknowns $T_{1\beta},T_{2\beta},n_{1\beta}$ and $n_{2\beta}$.

From Eq.(15), we have

$$n_{2\beta}\sqrt{T_{2\beta}}=n_{1\beta}\sqrt{T_{1\beta}}. \tag{20}$$

Substitution from Eqs.(14 and 20), with the help of Eq.(10), into Eq.(18), we obtain:

$$l\frac{\partial}{\partial\xi}\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)=$$
$$l\frac{\partial}{\partial\xi}\left(n_{2\beta}\sqrt{T_{2\beta}}\left(T_{1\beta}-T_{2\beta}\right)\right)=0 \tag{21}$$

Integrating Eq. (21), with respect to ξ, we obtain after factorization

$$\left(n_{1\beta}T_{1\beta}^{\frac{3}{2}}-n_{2\beta}T_{2\beta}^{\frac{3}{2}}\right)=\left(\begin{array}{c}n_{2\beta}\sqrt{T_{2\beta}}\left(\sqrt{T_{1\beta}}+\sqrt{T_{2\beta}}\right)\\\left(\sqrt{T_{1\beta}}-\sqrt{T_{2\beta}}\right)\end{array}\right) \tag{22}$$
$$=\theta_{1\beta}\theta_{2\beta}=C_{2\beta},$$

where , we put

$$\theta_{1\beta}=n_{2\beta}\sqrt{T_{2\beta}}\left(\sqrt{T_{1\beta}}+\sqrt{T_{2\beta}}\right),\theta_{2\beta}=\left(\sqrt{T_{1\beta}}-\sqrt{T_{2\beta}}\right) \tag{23}$$

and $C_{2\beta}$ is the integration constants. It's easy to show that $\theta_{1\beta},\theta_{2\beta}$ are constants, this comes from the assumption of the pressure uniformity since $P_{yy\beta}=\frac{n_{2\beta}}{2}\sqrt{T_{2\beta}}\left(\sqrt{T_{1\beta}}+\sqrt{T_{2\beta}}\right)=\frac{\theta_{1\beta}}{2}$ = constant, using Eq.(22), this implies that $\theta_{2\beta}$ is a constant as well [19,21].

For sampling the calculation, and for making the better usage of Eq. (20), we assumed a function $G_{\beta}(\xi)$ in the form:

$$G_{\beta}(\xi)=n_{2\beta}\sqrt{T_{2\beta}}=n_{1\beta}\sqrt{T_{1\beta}}. \tag{24}$$

From Eqs. (23 and 24) we can obtain:

$$T_{1\beta}(\xi)=\frac{\left(\theta_{1\beta}+\theta_{2\beta}G_{\beta}\right)^{2}}{4G_{\beta}^{2}},T_{2\beta}(\xi)=\frac{\left(\theta_{1\beta}-\theta_{2\beta}G_{\beta}\right)^{2}}{4G_{\beta}^{2}},$$
$$n_{1\beta}(\xi)=\frac{2G_{\beta}^{2}}{\left(\theta_{1\beta}+\theta_{2\beta}G_{\beta}\right)}\text{ and }n_{2\beta}(\xi)=\frac{2G_{\beta}^{2}}{\left(\theta_{1\beta}-\theta_{2\beta}G_{\beta}\right)}. \tag{25}$$

After performing some algebraic manipulations, we can integrate Eq. (19) with respect to ξ, with the help of Eqs. (14 and 22), obtained:

$$\frac{5}{4}l\left[\left(n_{1\beta}T_{1\beta}^{2}+n_{2\beta}T_{2\beta}^{2}\right)\right]+\frac{3}{2}lN_{\beta}\frac{\left(n_{1\beta}T_{1\beta}+n_{2\beta}T_{2\beta}\right)^{2}}{\left(n_{1\beta}+n_{2\beta}\right)}$$
$$=\left(\left(\frac{v_{\beta\beta}}{v_{AB}}+\frac{v_{\beta\alpha}}{v_{AB}}\right)\xi+m\right)\theta_{1\beta}\theta_{2\beta}+\theta_{3\beta}, \tag{26}$$

where $\theta_{3\beta}$ is the integration constant.

Substituting from Eqs. (25) into Eq. (26), yields:

$$\left(\frac{1}{1024G_{\beta}^{8}}\right)\left(\begin{array}{c}3lN_{\beta}\theta_{1\beta}^{9}-12lN\theta_{1\beta}^{7}\theta_{2\beta}^{2}G^{2}+\\18lN_{\beta}\theta_{1\beta}^{5}\theta_{2\beta}^{4}G_{\beta}^{4}+4l\theta_{1\beta}^{3}G_{\beta}^{6}\\(80-3N_{\beta}\theta_{2\beta}^{6})+G_{\beta}^{8}(-1024\theta_{3\beta}\\+\theta_{1\beta}\theta_{2\beta}(-1024m+960l\theta_{2\beta}+\\3lN_{\beta}\theta_{2\beta}^{7}+1024\left(\frac{v_{\beta\beta}}{v_{AB}}+\frac{v_{\beta\alpha}}{v_{AB}}\right)\xi))\end{array}\right)=0 \tag{27}$$

Solving it by the aid of symbolic software, we obtain eight roots for $G_{\beta}(\xi)$. We keep into consideration the root that preserves the positive signs of both temperature and concentration.

The values of the constants $\theta_{1\beta},\theta_{2\beta}$ and $\theta_{3\beta}$ can be estimated under the initial and boundary conditions (as $(y,t)=(0,0)\Rightarrow\xi=0$):

$$\frac{\left(n_{1\beta}(\xi=0)+n_{2\beta}(\xi=0)\right)}{2}=C_{\beta} \tag{28}$$

where $C_{\beta}=\frac{n_{s\beta}}{n_{s}}$ is the molar fraction.

$$\left(\frac{n_{1\beta}(\xi=0)T_{1\beta}(\xi=0)+n_{2\beta}(\xi=0)T_{2\beta}(\xi=0)}{n_{1\beta}(\xi=0)+n_{2\beta}(\xi=0)}\right)=1 \tag{29}$$

$$\left(n_{1\beta}(\xi=0)T_{1\beta}(\xi=0)^{\frac{1}{2}}-n_{2\beta}(\xi=0)T_{2\beta}(\xi=0)^{\frac{1}{2}}\right)=0 \tag{30}$$

The temperature of the incident particles is assumed to be T_2 while the temperature of the reflected particles from the plate is the temperature T_1, they are related such that[53,54]

$$T_{2\beta}(\xi=0)=\chi_{\beta}\,T_{1\beta}(\xi=0)\,:\,0<\chi_{\beta}\leq1, \tag{31}$$

where χ_β is the ratio between the plate and gas temperatures.

The parameter χ_β can take arbitrary positive value less than unity to guaranty that the plate is hotter than the gas.

We can obtain by solving the algebraic system of Eqs. (28-31) that

$$n_{1\beta}(\xi=0)=\left(2C_\beta-\frac{2C_\beta}{1+\sqrt{\chi_\beta}}\right),n_{2\beta}(\xi=0)=$$
$$\left(\frac{2C_\beta}{1+\sqrt{\chi_\beta}}\right),T_{1\beta}(\xi=0)=\left(\frac{1}{\sqrt{\chi_\beta}}\right)$$
$$\text{and }T_{2\beta}(\xi=0)=\left(\sqrt{\chi_\beta}\right). \tag{32}$$

The above four quantities represent the initial and boundary conditions.

By substituting from (32) into (23), to obtain

$$\theta_{1\beta}=2C_\beta\quad,\theta_{2\beta}=\chi_\beta^{\frac{-1}{4}}-\chi_\beta^{\frac{1}{4}}\quad, \tag{33}$$

then from (32) into (26),we got

$$\theta_3=\frac{1}{2\sqrt{\chi}}\left[\begin{array}{l}4mC_\beta(-1+\sqrt{\chi})\chi^{\frac{1}{4}}+\\ lC_\beta(5+(-5+3N)\sqrt{\chi}+5\chi)\end{array}\right] \tag{34}$$

By the way of introducing the obtained quantities T_1,T_2,n_1 and n_2 into the two stream Maxwellian distribution function;

$$\begin{cases}f_{1\beta}=\dfrac{n_{1\beta}}{T_{1\beta}^{\frac{3}{2}}}exp[\dfrac{-C^2}{T_{1\beta}}],\text{For }\ C_y>0\\[4mm] f_{2\beta}=\dfrac{n_{2\beta}}{T_{2\beta}^{\frac{3}{2}}}exp[\dfrac{-C^2}{T_{2\beta}}],\text{For }\ C_y<0\end{cases} \tag{35}$$

we can get the sought distribution functions. These estimated distribution functions of the gas particles enable one to study their behavior in the investigated system, which is not possible by taking the way of the solution of Navier–Stokes equations. This will be the starting point to predict the irreversible thermodynamic behavior of the system in the next section.

3. The Non-Equilibrium Thermodynamic Properties of the System

The everyday resorts to the linear theory of the thermodynamics of irreversible processes still constitute great interests [55-60]. This is associated both with the general theoretical importance of this theory and its numerous applications in various branches of science. It is unquestionable that the concept of entropy has played an essential role both in the physical and biological sciences [61]. Thus, we start the thermodynamic investigations of the problem from the evaluation of the entropy S per unit mass of the binary gas, which is written as [62]:

$$S(y,t)=\rho^{-1}\sum_{\beta=A}^{B}\rho_\beta S_\beta \tag{36}$$

where S_β is the entropy of the gas species of the β type. It is denoted by[63]:

$$S_\beta(y,t)=-\int f_\beta Logf_\beta d\underline{c}\ =\frac{\pi^{\frac{3}{2}}}{8}\left(\begin{array}{l}n_{1\beta}\left(3-4\ln\left(\dfrac{n_{1\beta}}{T_{1\beta}^{\frac{3}{2}}}\right)\right)\\[4mm]+n_{2\beta}\left(3-4\ln\left(\dfrac{n_{2\beta}}{T_{2\beta}^{\frac{3}{2}}}\right)\right)\end{array}\right).$$

The y-component of the entropy flux vector has the form [64]:

$$J_y(y,t)=\sum_{\beta=A}^{B}J_{y\beta}(y,t) \tag{37}$$

where $J_{y\beta}$ is the y-component of the entropy flux of the gas species of the β type. It is denoted by :

$$J_{y\beta}(y,t)=-\int c_y f_\beta Logf_\beta d\underline{c}=\frac{\pi}{2}\left(\begin{array}{l}n_{1\beta}\sqrt{T_{1\beta}}\left[1-Ln\left(\dfrac{n_{1\beta}}{T_{1\beta}^{\frac{3}{2}}}\right)\right]-\\[4mm]n_{2\beta}\sqrt{T_{2\beta}}\left[1-Ln\left(\dfrac{n_{2\beta}}{T_{2\beta}^{\frac{3}{2}}}\right)\right]\end{array}\right)$$

The Boltzmann's entropy production [56-60] in the unsteady state σ_S is expressed as:

$$\sigma_S(y,t)=\frac{\partial S(y,t)}{\partial t}+\underline{\nabla}\ \cdot\ \underline{J}\ (y,t) \tag{38}$$

Following the general theory of thermodynamic [65-70], we could estimate the thermodynamic forces; the first corresponding to the change in the concentration X_n, as:

$$X_n(y,t)=\frac{\Delta y}{n(y,t)}\frac{\partial n(y,t)}{\partial y}, \tag{39}$$

The second thermodynamic force corresponding to the change in the temperature X_T, as:

$$X_T(y,t)=\frac{\Delta y}{T(y,t)}\frac{\partial T(y,t)}{\partial y}. \tag{40}$$

The third thermodynamic force corresponds to the change in the radiation field energy X_R, as:

$$X_R(y,t)=\frac{\Delta y}{U_R(y,t)}\frac{\partial U_R(y,t)}{\partial y}, \tag{41}$$

where $U_{R\beta}(y,t) = \left(\dfrac{16\sigma T_s^{\,4}}{3cn_s K_B T_s}\right) T_\beta^{\,4}(y,t)$ is the dimensionless

radiation field energy influences the gas particles and Δy is the thickness of the layer adjacent to the flat plate in units of the mean free path-the distance between two collisions of the gas particles-in dimensionless form.

After calculating the thermodynamic forces and the entropy production, we can obtain the kinetic coefficients Lij from the relationship between the entropy production and the thermodynamic forces, via the form [58, 59]:

$$\sigma_S(y,t) = \sum_i \sum_j L_{ij} X_i X_j$$
$$= (X_1 \quad X_2 \quad X_3)$$
$$\begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \geq 0 \qquad (42)$$

This constituted the restriction on the sign of phenomenological coefficients L_{ij}, which arise as a consequence of the second law of the thermodynamics [58], which can be deduced from the standard results in algebra. The necessary and sufficient conditions for $\sigma_s(y,t) \geq 0$ are fulfilled by the determinant

$$\left| L_{ij} + L_{ji} \right| \geq 0 \qquad (43)$$

and all its principal minors are non-negative too. Another restriction on L_{ij} was established by Onsager (1931). He found that, besides the restriction on the sign, the phenomenological coefficients verify important symmetry properties. Invoking the principle of microscopic reversibility and using the theory of fluctuations, Onsager was able to demonstrate the symmetry property that is denoted by,

$$L_{ij} = L_{ji} \qquad (44)$$

which is called the Onsager's reciprocal relations.

The Gibbs formula for the variation of the internal energy applied to the system (binary gas mixture + heated plate), $dU(y,t)$ is

$$dU(y,t) = dU_S(y,t) + dU_V(y,t) + dU_R(y,t) \qquad (45)$$

The internal energy change due to the variation of the extensive variables, such as entropy dU_S, volume dU_V and in addition the temperature gradient produced by the thermal radiation field dU_R, are respectively read for a binary gas [62], as follows:

$$dU_S(y,t) = \rho^{-1} \sum_{\beta=A}^{B} \rho_\beta dU_{S\beta} \ ,$$

where

$$dU_{S\beta}(y,t) = T_\beta dS_\beta, \qquad (46)$$

$$dU_V(y,t) = \rho^{-1} \sum_{\beta=A}^{B} \rho_\beta dU_{V\beta}, \text{ where}$$

$$dU_{V\beta}(y,t) = -P_\beta dV_\beta, \qquad (47)$$

$$dU_R(y,t) = \rho^{-1} \sum_{\beta=A}^{B} \rho_\beta dU_{R\beta}, \text{ where}$$

$$dU_{R\beta}(y,t) = \omega_\beta \frac{\partial T_\beta^{\,4}(y,t)}{\partial y} \Delta y \ , \ \ \omega_\beta = \frac{16\sigma T_s^{\,4}}{3cn_s K_B T_s}. \qquad (48)$$

The pressure and change in volume are $P_\beta = n_\beta\, T_\beta$ and $dV_\beta = -\dfrac{dn_\beta}{n_\beta^{\,2}}$ respectively, and

$dn = \dfrac{\partial n}{\partial t}\delta t + \dfrac{\partial n}{\partial y}\delta y$, $dS = \dfrac{\partial S}{\partial t}\delta t + \dfrac{\partial S}{\partial y}\delta y$ and $\delta y = 1, \delta t = 1$.

4. Results and Discussion

We investigated the behavior of the binary gas mixture under the influence of a non-linear thermal radiation field in the unsteady state of a plane heat transfer problem in the system (binary neutral rarefied gas mixture + heated plate). The thermal radiation is introduced, in the force term of the Boltzmann equation for a rarefied gas subject to [71] the inequalities $\lambda \gg \delta \gg d$, where $\delta = n^{-1/3}$ is the average spacing between the molecules, both λ and d are the mean free path and the molecular diameter, respectively. In all calculations and figures, we take the following parameters values from [72, 73] for the Argon-Neon binary gas mixture, where the particles of the A and B components are the argon and neon gases, respectively:

$T_s = 1000K; \sigma_s = 5.6705 \times 10^{-5}$ J.m^{-2}.sec^{-1}.K^{-4};

$R = 8.3145$ J.K^{-1}.mol^{-1}; $K_B = 1.3807 ? \quad ^{-23}10^{-1}$J.

$c = 2.9979 \times 10^8$ m.sec^{-1}; $n_s = 3 \times 10^{18}$ m^{-3};

$m_A = 39.948 m_u$; $m_B = 20.183 m_u$;

$m_u = 1.6605 ? \ 10^{-27}$ Kg is atomic mass unit;

$d_A = 3.84 \times 10^{-10}$ m; $\lambda_A = 1.017$ m;

$V_{TA} = 6.45 \times 10^2$ m.sec^{-1}; $\tau_A = 1.577 \times 10^3$ sec;

$d_B = 2.425 \times 10^{-10}$ m; $\lambda_B = 2.551$ m;

$V_{TB} = 9.077 \times 10^2$ m.sec^{-1}; $\tau_B = 2.81 \times 10^3$ sec;

for a fixed $C_A = 0.5$; $N_A(1000K) = 0.609694$,

$N_B(1000K) = 1.20676$,

where K_B, $V_{T\beta}$ and τ_β are Boltzmann constant, thermal velocity, and the relaxation time of the gas species of the β type, respectively.

Although we calculate all the sought variables in three various radiation field intensities due to different plate temperatures ($T = 800K, 1000K, 1400K$), we particularize

our graphics in one case corresponding to ($T = 1000K$), to economize the figures and illustrations.

All figures show that all variables satisfy the equilibrium condition where the variables are in the equilibrium (as $\chi = 1$) as follows:

$$n(\chi = 1) = 1,\ T(\chi = 1) = 1,\ S(\chi = 1) =$$
$$It's \text{ maxmum value},\ \sigma(\chi = 1) = 0, dU_S(\chi = 1) = 0,$$
$$dU_V(\chi = 1) = 0,\ dU_R(\chi = 1) = 0,\ X_T(\chi = 1) = 0,$$
$$X_n(\chi = 1) = 0,\ X_R(\chi = 1) = 0,$$

see figures (1 to 10-a).
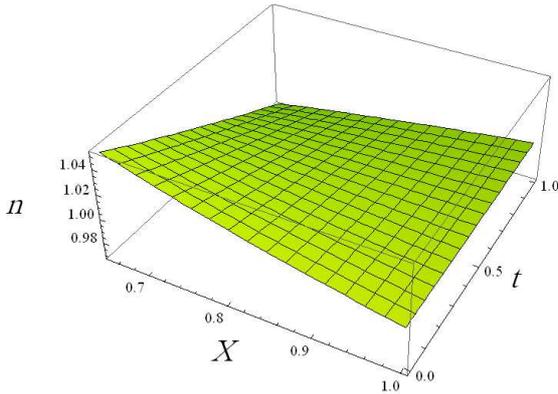
Figure (1-a). Concentration n vs. t and χ at C_A =0.5, y=0.3.
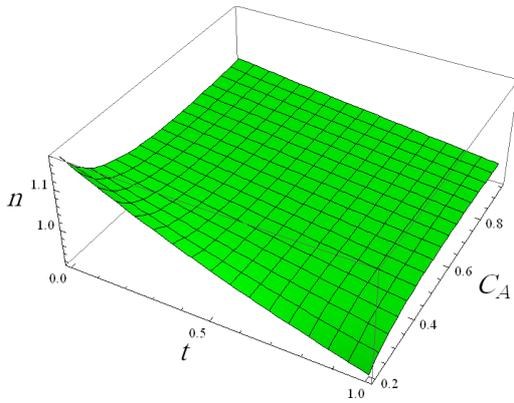
Figure (1-b). Concentration n vs. t and C_A at χ=0.66, y=0.3.
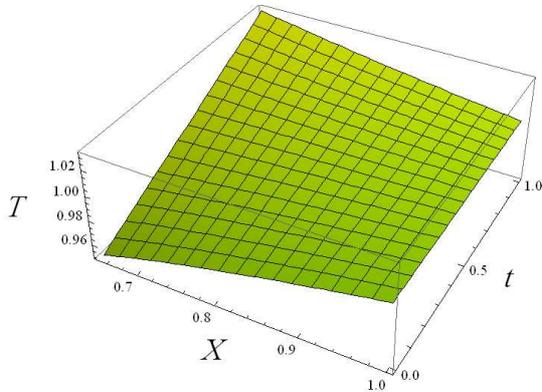
Figure (2-a). Temperature T vs. t and χ at C_A =0.5, y=0.3.
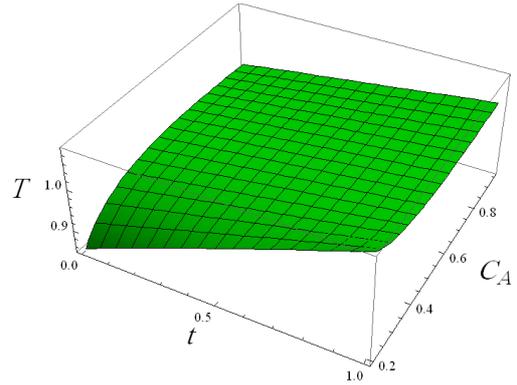
Figure (2-b). Temperature T vs. t and C_A at χ=0.66, y=0.3.
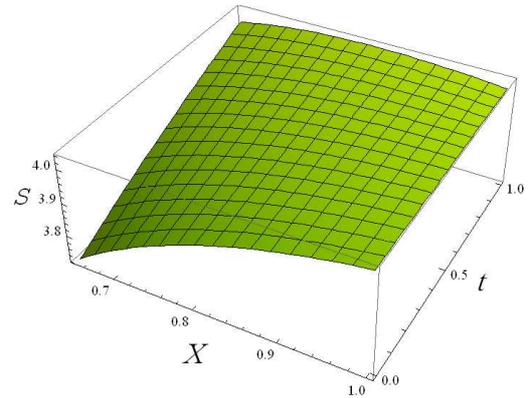
Figure (3-a). Entropy S vs. t and χ at C_A =0.5, y=0.3.
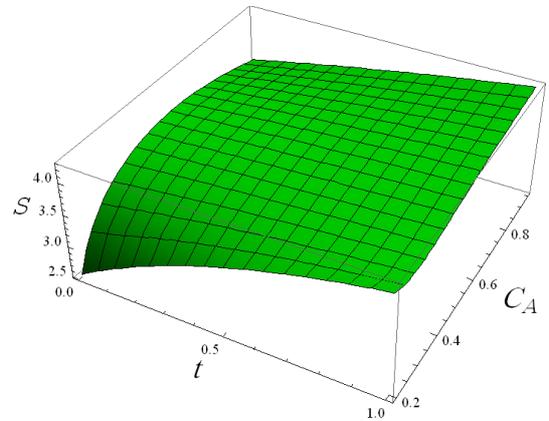
Figure (3-b). Entropy S vs. t and C_A at χ=0.66, y=0.3.
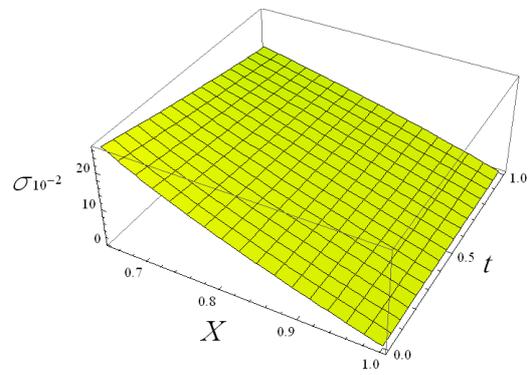
Figure (4-a). Entropy production σs vs. t and χ at C_A =0.5, y=0.3.
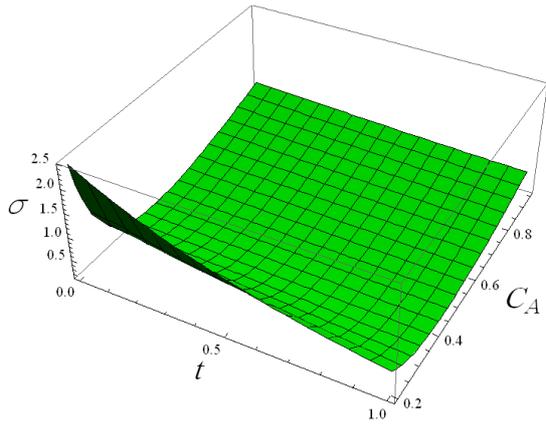
Figure (4-b). *Entropy production σs vs. t and C_A at χ=0.66, y=0.3.*
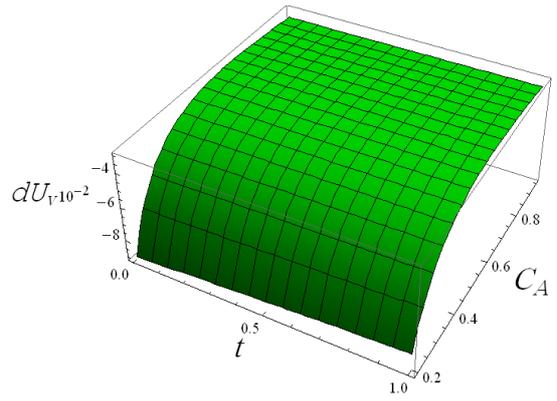
Figure (6-b). *dU_V vs. t and C_A at χ=0.66 , y=0.3.*
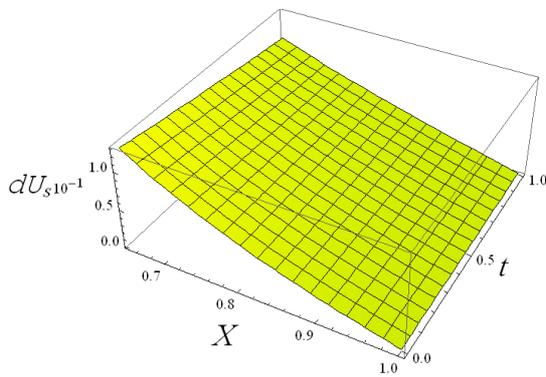

Figure (5-a). *dU_S vs. t and χ at C_A =0.5, y=0.3.*
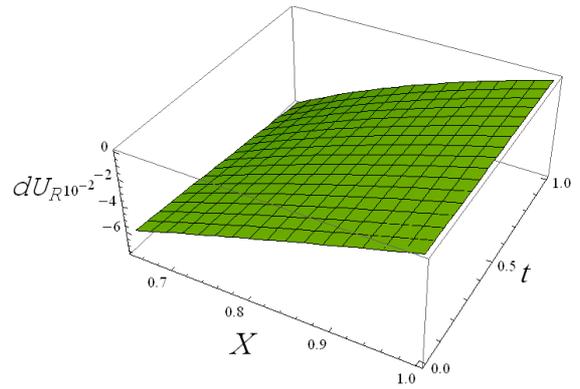
Figure (7-a). *dU_R vs. t and χ at C_A =0.5, y=0.3.*
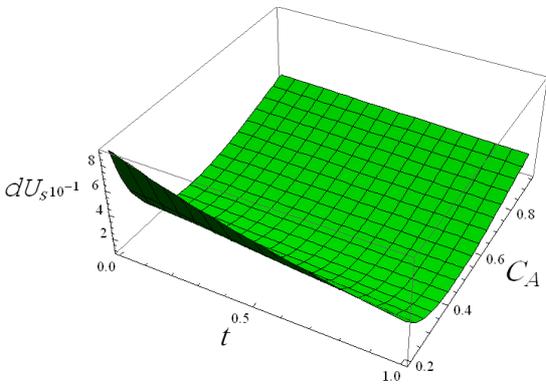
Figure (5-b). *dU_S vs. t and C_A at χ=0.66 , y=0.3.*
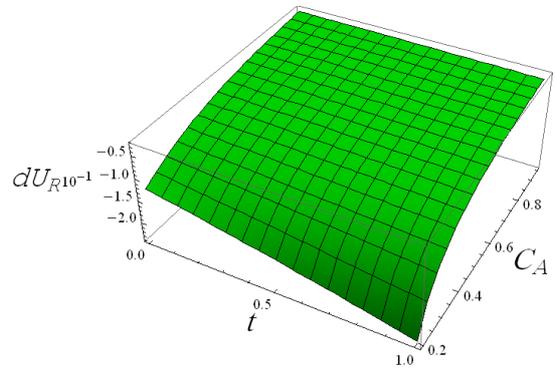
**Figure (7-b).** *dU_R vs. t and C_A at χ=0.66 , y=0.3.*
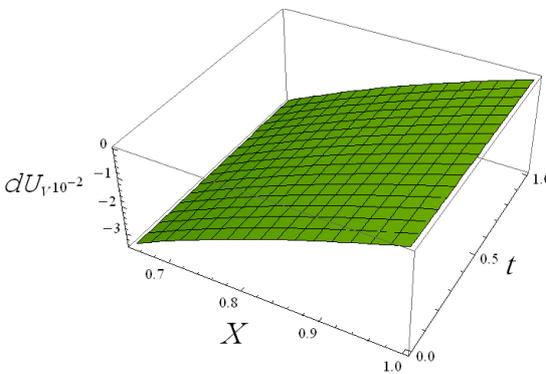
**Figure (6-a).** *dU_V vs. t and χ at C_A =0.5, y=0.3.*
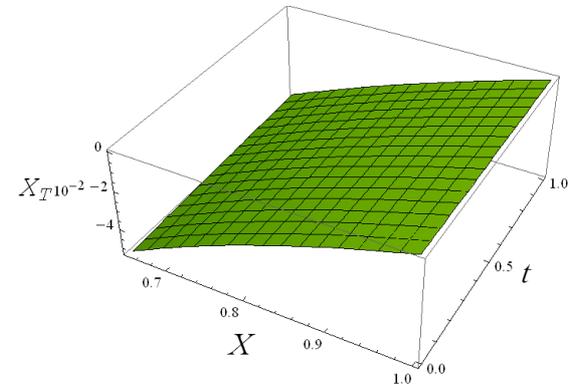
Figure (8-a). *Thermodynamic force X_T vs. t and χ at C_A =0.5, y=0.3.*
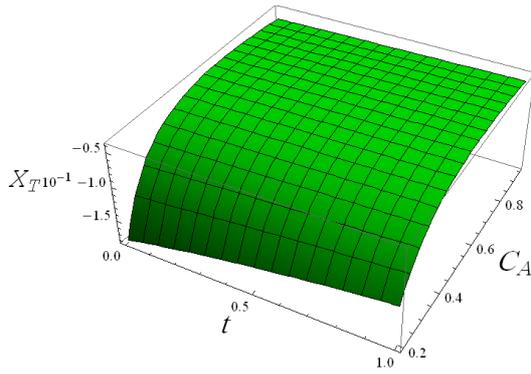
**Figure (8-b).** *Thermodynamic force X_T vs. t and C_A at χ=0.66 , y=0.3.*
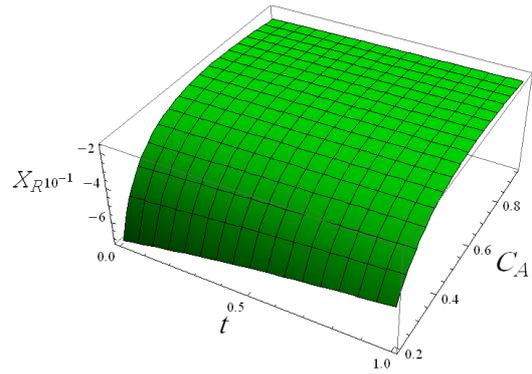
Figure (10-b). *Thermodynamic force X_R vs. t and C_A at χ=0.66, y=0.3.*
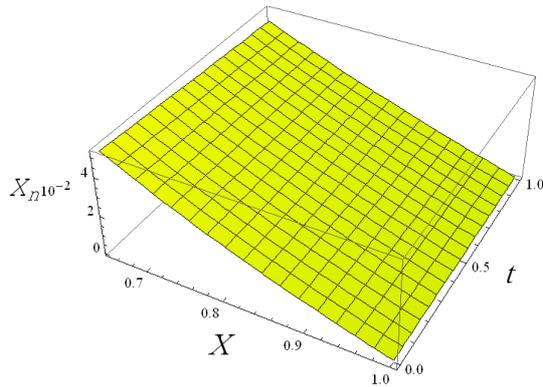

Figure (9-a). *Thermodynamic force X_n vs. t and χ at C_A =0.5, y=0.3.*
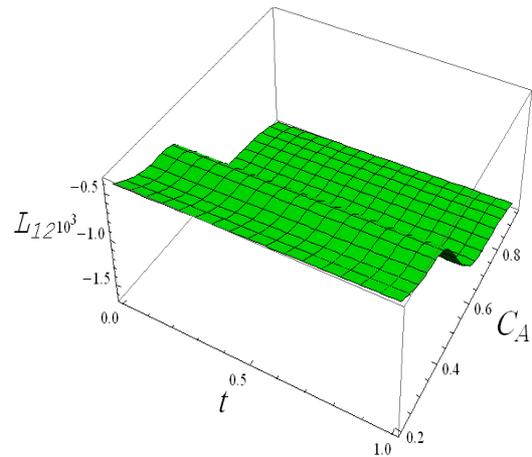

Figure (11-a). *Kinetic coefficient $L_{12} \equiv L_{21}$ vs. t and C_A at χ=0.66, y=0.3.*
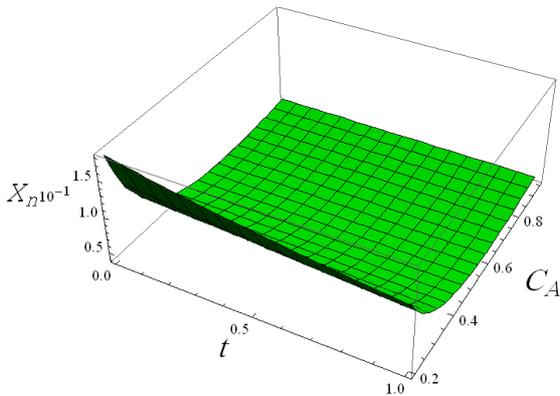

**Figure (9-b).** *Thermodynamic force X_n vs. t and C_A at χ=0.66 , y=0.3.*
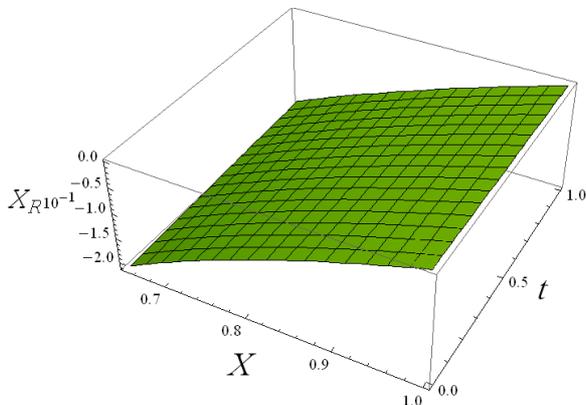
Figure (10-a). *Thermodynamic force X_R vs. t and χ at C_A =0.5, y=0.3.*

Now, we will discuss the behavior of the gas particles far from the equilibrium state; while the number density *n* decreases, with increasing time, the temperature *T* increases, these happen for all values of χ, this is due to the fact of the uniform pressure, see figures (1,2-a). Similarly, while the number density *n* increases, with increasing time, the temperature *T* decreases, these happen for all values of C_A, this is due to the fact of the uniform pressure, see figures (1,2-b). It is shown from figures (3,4-a,b) that the entropy *S* always increases with time and the entropy production σ_S has a nonnegative values for all values of *t* , χ and the molar fraction C_A. This gives a complete satisfaction of the second law of thermodynamics, the Boltzmann H-theorem and this behavior agrees with the famous Le-Chatelier principle.

The behavior of the different contributions of the internal energy change can be illustrated in figures (5, 6, 7-a,b). The numerical ratios between the different contributions of the internal energy changes based upon the total derivatives of the extensive parameters are predicted via the Gibbs formula. Taking into consideration their different tendencies, the maximum numerical values of the three contributions at various radiation field intensity (corresponding to various plate temperatures), are ordered in magnitude as follows:

(a) For a fixed value $C_A = 0.5$ and variable values of χ in the considered range ($0.65 < \chi < 1$), see figures (5-a, 6-a, 7-a), we had:.

$$dU_S(1000) : dU_V(1000) : dU_R(1000) \sim 1 : 0.26 : 0.5$$
$$dU_S(1400) : dU_V(1400) : dU_R(1400) \sim 1 : 0.33 : 3$$

Figure (5-a) indicated that, dU_R reaches its maximum numerical value at $\chi = 0.65$ which means that the more the temperature difference, between the temperature of the plate surface and the temperature of the binary gas mixture particles, the more will be the effective contribution of the thermal radiation energy in the total energy change of the system. This is attributed to the behavior of the corresponding values of the thermodynamic force X_R itself, see figure (10-a).

(b) For a fixed value ($\chi = 2/3$) and variable values of C_A in the considered range ($0.2 < C_A < 0.95$), figures (5-b, 6-b, 7-b), we had:

$$dU_S(1000) : dU_V(1000) : dU_R(1000) \sim 1 : 0.1 : 0.297$$
$$dU_S(1400) : dU_V(1400) : dU_R(1400) \sim 1 : 0.125 : 1.4$$

The contribution of dU_R reaches its maximum numerical value at the values of $C_A = 0.2$ and $C_B = 0.8$ which represent indirectly the concentrations of the heavier gas (Argon), and the lighter gas (Neon) respectively. This means that, the more the lighter mass of the gas species, the more the effectiveness of the thermal radiation energy contribution in the total energy change of the system, see figure (5-b).This is also due to the behavior of the corresponding value of the thermodynamic force X_R itself corresponding to each component in the binary gas mixture, see figure (10-b).


Figure (12-a). Kinetic coefficient $L_{23} \equiv L_{32}$ vs. t and C_A at χ=0.66, y=0.3.
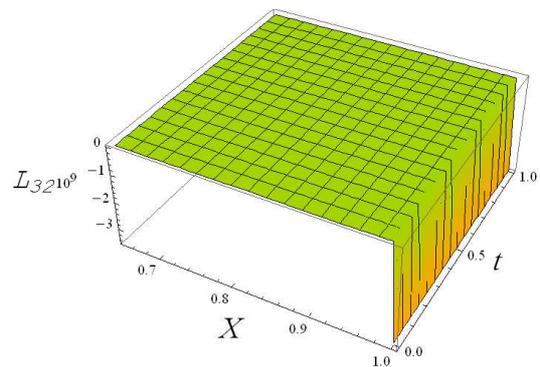
Figure (12-b). Kinetic coefficient $L_{23} \equiv L_{32}$ vs. t and χ at C_A =0.5, y=0.3.
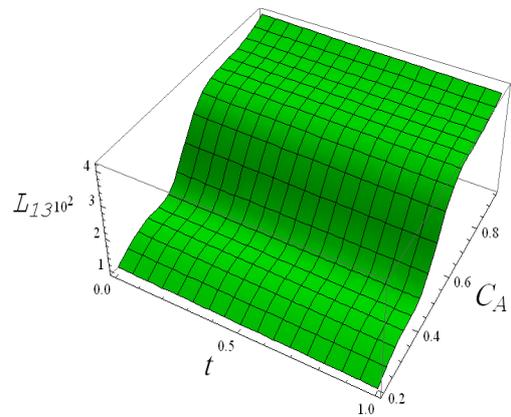

Figure (13-a). Kinetic coefficient $L_{31} \equiv L_{13}$ vs. t and C_A at χ=0.66, y=0.3.

Figure (11-b). Kinetic coefficient $L_{12} \equiv L_{21}$ vs. t and χ at C_A =0.5, y=0.3.

The thermodynamic force due to the gradient of temperature X_T will have the opposite direction to the thermodynamic force due to the gradient of the density X_n, see figures (8,9-a,b) .This gives a qualitative agreement in the behavior studied in [71], comparing with the same (Ar-Ne) mixture at the same molar fraction $C_A = 0.5$.

According to our calculations, the restrictions imposed on the kinetic coefficients L_{ij} are satisfied where $L_{11} \geq 0$, $L_{22} \geq 0$ and $L_{33} \geq 0$, for all values in the considered ranges of the taken values for both of χ and C_A .The celebrated Onsager's reciprocal relations are satisfied, where we have $\left(L_{12} \equiv L_{21}, L_{13} \equiv L_{31} \text{ and } L_{32} \equiv L_{23}\right)$, for all values in the considered ranges for χ and C_A , see figures (11,12,13-a,b).

**Figure (13-b).** Kinetic coefficient $L_{31} \equiv L_{13}$ vs. t and χ at C_A =0.5, y=0.3.

5. Conclusions

By analyzing the results, we conclude that:

a) The active controlling factors, which are directly proportional to the effectiveness of the radiation energy in the total energy change of the system, are:

b) The ratio between the temperature of the plate surface and the gas mixture particles, ii) The mass ratio between each component of the binary gas mixture, iii) The molar fraction of each component of the mixture.

c) At a relatively high temperature ($T \geq 1000K$), the radiation energy contribution in the total internal energy change becomes the dominated one and cannot be ignored at all.

d) At a relatively small temperature ($T < 600K$), the radiation energy contribution in the total internal energy change, in the considered system, become less by orders of magnitude than the other kinds of energy contributions.

e) The lighter gas component (Neon) of the binary gas mixture is affected by the non-linear thermal radiation field more than for the heavier one (Argon).

f) The second law of thermodynamics, the Boltzmann H-theorem, the Le-Chatelier principle, the Onsager's inequality and the Onsager's reciprocal relations, are all satisfied for the studied binary mixture system.

g) The negative sign at some of the kinetic coefficients, corresponding to cross effects, imply in these cases that there is a heat flux opposite to the main flux due to the imposed thermodynamic force (gradient). For example, the negative sign in front of $L_{Tn} = L_{12}$ and $L_{Rn} = L_{13}$, implies that there is a flow caused by the temperature gradient, from a lower to a higher temperature, known as thermal diffusion (or Soret effect) which gives a qualitative agreement with the study [71].

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