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Realistic Simulations of Non-Linear Acceleration of the Rocket in the Air

Haoyuan Xiong

Beijing Haidian Foreign Language Shiyan School, Beijing, China

Email address:
m13581520966@gmail.com

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Abstract: The motion of the object in the medium has always been a hot research topic, and it is closely connected with many applications in our life. The acceleration of the object with multiple forces becomes very complicated, especially when these forces depend on the motion of the object. The exact formula for the object motion is a differential-integral equation and is very difficult to be solved analytically. One example of this kind of motions is the rocket launch. With sufficient thrust, the rocket can obtain an acceleration large enough to escape from the gravity of the earth. With the increasing height, the gravity from the earth becomes smaller, which affects the net acceleration of the rocket. Meanwhile, the air resistance becomes more and more important when the velocity of the rocket increases. It even plays the main role in the middle stage of the launch. Also, as the air resistance depends on both the velocity of the rocket and the air density (there is no air resistance in vacuum), the air resistance will decrease when the air density becomes small enough at the large height. In this article, a model that includes all of the factors mentioned above is established, and how these forces change the velocity of the rocket is analyzed. Two scenarios, one with air resistance and one without, are described. The velocity of the rocket in each scenario is represented by graphs, which are compared. With justification, the Taylor series is used to solve the differential-integral equation, and it is found that the fuel thrust and the gravity become important in the rocket launch at the beginning stage. In the middle stage, the air resistance begins to have a significant effect and reduces the acceleration of the rocket. In the final stage, there is virtually no gravity or air resistance, and only the fuel thrust contributes to the acceleration of the rocket.

Keywords: Non-linear Acceleration, Taylor Expansion, Rocket Launch, Air Resistance, Gravity

1. Introduction

The acceleration process is very common in nature. It ranges from the acceleration of racing cars and the flow of water on mountains, to the launch of rockets. This process relates to both the state of the motion and the force acting on the object. When the forces on the object change with the status of the object (such as the air resistance), the question becomes especially complicated. Furthermore, when the mass of the object changes with time, the acceleration will also be different even with the same force (such as a missile launch). In the article, the rocket launch which relates to all of these factors is investigated. These phenomena are easily observed, but it is hard to completely describe these kinds of acceleration processes by utilizing physical models or mathematical equations. This article focuses on a specific model, the launch of a rocket, and intends to set up a more realistic model. More specifically, this paper introduces many factors, including gravitational acceleration, air resistance, as well as the motion of a variable mass object to solve the problem of the acceleration of a rocket [1-3].

The general study of the forces on a rocket belongs to the field of ballistics. Spacecrafts are further studied in the subfield of astrodynamics. Flying rockets are primarily affected by the following: thrust from the engines; gravity from celestial bodies; drag if moving in the atmosphere [4-6]. Rockets that must travel through the air are usually tall and thin as this shape gives a high ballistic coefficient and minimizes drag losses. In addition, the inertia and centrifugal pseudo-force can be significant due to the path of the rocket around the center of a celestial body; when high enough speeds in the right direction and altitude are achieved, a stable orbit or escape velocity is obtained.

The first factor introduced is gravitational acceleration. In physics, gravitational acceleration is the acceleration of an object caused by the force of gravity. Neglecting friction such as air resistance, all small bodies accelerate in a gravitational field at the same rate relative to the center of mass.[7-9] This is true regardless of the masses or compositions of the bodies. At different points on Earth, objects fall with an acceleration between 9.764 m/s^2 and 9.834 m/s^2 depending on altitude and latitude, with a conventional standard value of 9.80 m/s^2 (approximately 32.174 ft/s^2) [10]. This does not take into account other effects, such as buoyancy and drag.

In the following parts of this article, the change of acceleration during the accelerating process of a rocket is taken into consideration, which makes it a more realistic model.

In the meantime, the rocket is subjected to drag. Drag is a force opposite to the direction of the rocket's motion. This decreases acceleration of the vehicle and produces structural loads. Deceleration force for fast-moving rockets is calculated using the drag equation. Drag can be minimized by an aerodynamic nose cone, by using a shape with a high ballistic coefficient (the "classic" rocket shape—long and thin), and by keeping the rocket's angle of attack as low as possible. During a rocket launch, as the vehicle speed increases, and the atmosphere thins, there is a point of maximum aerodynamic drag called Max Q. This determines the minimum aerodynamic strength of the vehicle, as the rocket must avoid buckling under these forces.[10]

Finally, the motion of a variable-mass object is introduced. A variable mass is an object whose mass changes significantly during the movement. This motion is called the motion of a variable-mass system. The change in mass of the object referred to here does not mean the elimination or production of mass but that a part of the mass of the object is not considered before or after a certain instant. The former is equivalent to the mass of the object incorporated by mass, and the latter is equivalent to mass separation and the mass of the object is reduced. If the rotational speed and acceleration of the variable mass are negligible compared to the translational velocity and acceleration, the variable mass can be regarded as a variable mass point. The problem of variable mass in this case is still in the category of classical mechanics.

There are many examples of variable mass in engineering and nature. The rocket burns out the fuel during the process of accelerating, which changes the mass of the rocket. In general, the jet is a variable-mass object, because it not only draws in air continuously to increase the mass but also ejects gas to reduce the mass, that is, the incorporation and separation of mass occur simultaneously. The mass of the meteoroid is reduced by frictional combustion when it enters the atmosphere. The mass of the ice floes increases due to the freezing of seawater or decreases due to melting; The mass and moment of inertia of the spindles of the cotton mill are constantly changing during the rotation, and so on.

Modern rockets use the method of gradually ejecting the burned gas outward to increase the speed of the rocket itself. Therefore, this model obviously belongs to the problem of a variable-mass system.

The following parts of the essay simulate the motion of a rocket by adopting Newton's second law. During the calculation, mathematical skills like numerically solving differential equations and Taylor series are used. Many factors mentioned above are considered, and the results are analyzed.

2. Theoretical Model

In this model, 40000m is chosen as the final height of the orbit that rockets enter.

Three factors that introduce complications to the problem are taken into consideration in the analysis [11].

The first factor is gravitational acceleration(g). The value of g decreases as the rocket gradually gains height, so the function g(h) is used to account for this change. For the sake of simplicity, the value of g is supposed to be inversely proportional with the height of the rocket. The value of gravitational acceleration at any moment during the launching process is decided by the following equation, where g_0 is the gravitational acceleration on the ground and has a value of $10m/s^2$.

$$g(h) = g_0 - g_0(1 - \frac{h}{h_f})$$

In the meantime, the change in air density is not negligible. The rocket is subjected to air resistance, which is related to air density, a value that decreases linearly as the rocket gains height and infinitesimally approaches zero at H_f, the final height of the rocket. The following equation decides f (air resistance force). Air resistance $f = Cv^2 \times \frac{\rho_{air}}{\rho_{ground}}$ [12-13], where C is a constant that has a value of 10. Therefore, the air resistance can be expressed in the following form.

$$f = Cv^2 \times (1 - \frac{h}{H_f})$$

Finally, the rocket loses mass as fuel is expelled. A typical value of $\frac{dm}{dt} = 50kg/s$ is chosen in this essay. The following equation decides the thrust (F) of the rocket. A is a constant with a value of 5500, and the initial mass is assumed to be 15000 kg.

$$F = A\frac{dm}{dt}$$

The analysis of three factors such as upward thrust from the fuel, gravity, and air resistance is broken down into two parts.

In order to check the effects of each factor on the acceleration of the rocket, scientists consider each factor separately. In the first scenario, the effect of air resistance is excluded, and the net force of the rocket is thus comprised of two forces: the upward thrust force and the downward gravitational force. The net force thus could be expressed in the form below. F decides the thrust provided by the fuel of the rocket [14-17].

$$m\frac{dv}{dt} = F - mg(h)$$

$$m\frac{dv}{dt} = F - m\left(1 - \frac{h}{H_f}\right)g_0$$

$$\mathrm{m}\frac{dv}{dt} = F - m(1 - \frac{1}{H_f}\int_0^t vdt)g_0$$

In order to calculate the result, the equation above is rewritten in a discrete form which can be solved numerically. This expression could then be calculated in Excel or C++ codes.

$$m_n\frac{v_{n+1} - v_n}{\Delta t} = \frac{dm}{dt}A - m_n(1 - \frac{1}{H_f}\sum_0^n v_n\Delta t)g_0$$

To account for the effect of air resistance, just subtract from the net force above a term that expresses air resistance. Written in a discrete form, the equation in the second scenario that includes the effect of air resistance is shown below.

$$m_n\frac{v_{n+1} - v_n}{\Delta t} = \frac{dm}{dt}A - m_n(1 - \frac{1}{H_f}\sum_0^n v_n\Delta t)g_0 - Cv_n^2(1 - \frac{1}{H_f}\sum_0^n v_n\Delta t)$$

In the expression above n indicates the steps. The length of a time step is 1s. When calculating the distance traveled by the rocket, it is assumed that in each time interval the amount of lost mass is negligible and that the acceleration is negligible. This is acceptable if the time interval is small enough, and there's thus no quadratic term that accounts for the acceleration. The following calculations are conducted to demonstrate this. The scenario without air resistance is used.

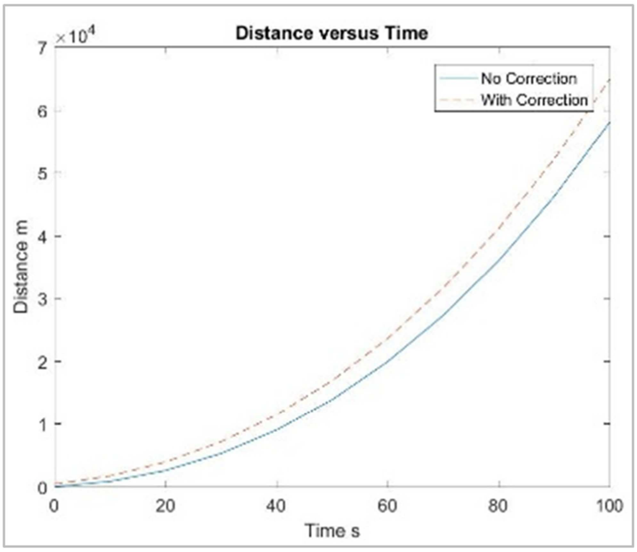

Figure 1. *The distance rocket travel as a function of the time. Time interval in the numerical discrete form is taken as 10 seconds.*

The two graphs (Figure 1 and 2) above illustrate this concept. When the time interval equals 1s, the difference between the two results is negligible, and the sum in the discrete is closer to the continuous integral. while the graph with the time step to be 10 seconds, exhibits great disagreements between two results. Because the approximation of the "constant velocity" for the rocket in each time step is not a good approximation anymore. In other words, the rocket velocity at the beginning of one step is different from the velocity after the 10 seconds with non-negligible magnitude.

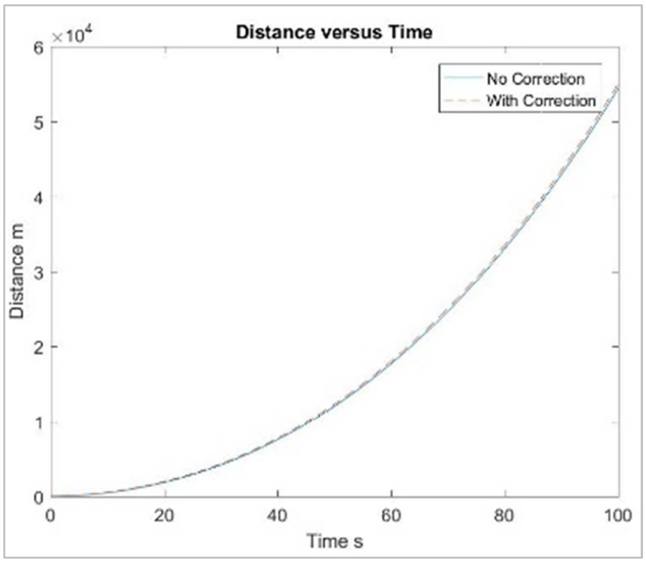

Figure 2. *The distance rocket travel as a function of the time. Time interval in the numerical discrete form is taken as 1 seconds.*

The analysis here adopts the method of Taylor Series, a representation of the value of an infinitely differentiable function as a sum of infinite terms calculated form the values of the function's derivatives. Taylor series takes the following form.

$$f(x_1) = \sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x_1 - x_0)^n$$

If the value of the function at x_a is known, the value at x_{a+1} can be approximated. The more terms added, the more Taylor series resembles the original function. If, ideally, an infinite number of terms is added, the Taylor series perfectly reproduces the original function. In this model, when calculating the distance traveled by the rocket, the Taylor series takes a familiar form.

$$d(t_1) = \sum_{n=0}^{\infty}\frac{d^{(n)}(t_0)}{n!}(t_1 - t_0)^n$$

$$\mathrm{d}(t_1) = d(t_0) + d'(t_0)(t_1 - t_0) + \frac{d''(t_0)}{2!}(t_1 - t_0) + \cdots$$

The zeroth derivative of distance is just distance itself, the first derivative is the velocity, and the second derivative is the acceleration. Thus, the Taylor series can be expressed in a familiar form, and this is the expression used in the production of the data,

$$\mathrm{d}(t_1) = d(t_0) + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 + \cdots$$

The time step refers to $t_1 - t_0$. When the time interval is greater than one, the quadratic term magnifies the effect of acceleration. On the other hand, as the time interval decreases, the effect of acceleration becomes less significant. When a positive number smaller than 1 is raised to the second power, it becomes smaller and therefore can be neglected within some precision.

3. Numerical Results and Analysis

In the real analysis of velocity, for the sake of precision, a time step of 1s is used in the model, and the method of quadratic term correction term is not adopted.

In the first scenario, the effect of air resistance is neglected.

Figure 3 is a smooth curve. The thrust is constant throughout the launching process, but the gravitational force decreases as the rocket gains height. This explains why the curve as an increasing slope. At the later time period, the height becomes larger, and the gravity constant becomes smaller. The net acceleration constant of the rocket will become larger, which is indicated by the slope of the velocity in Figure 3.

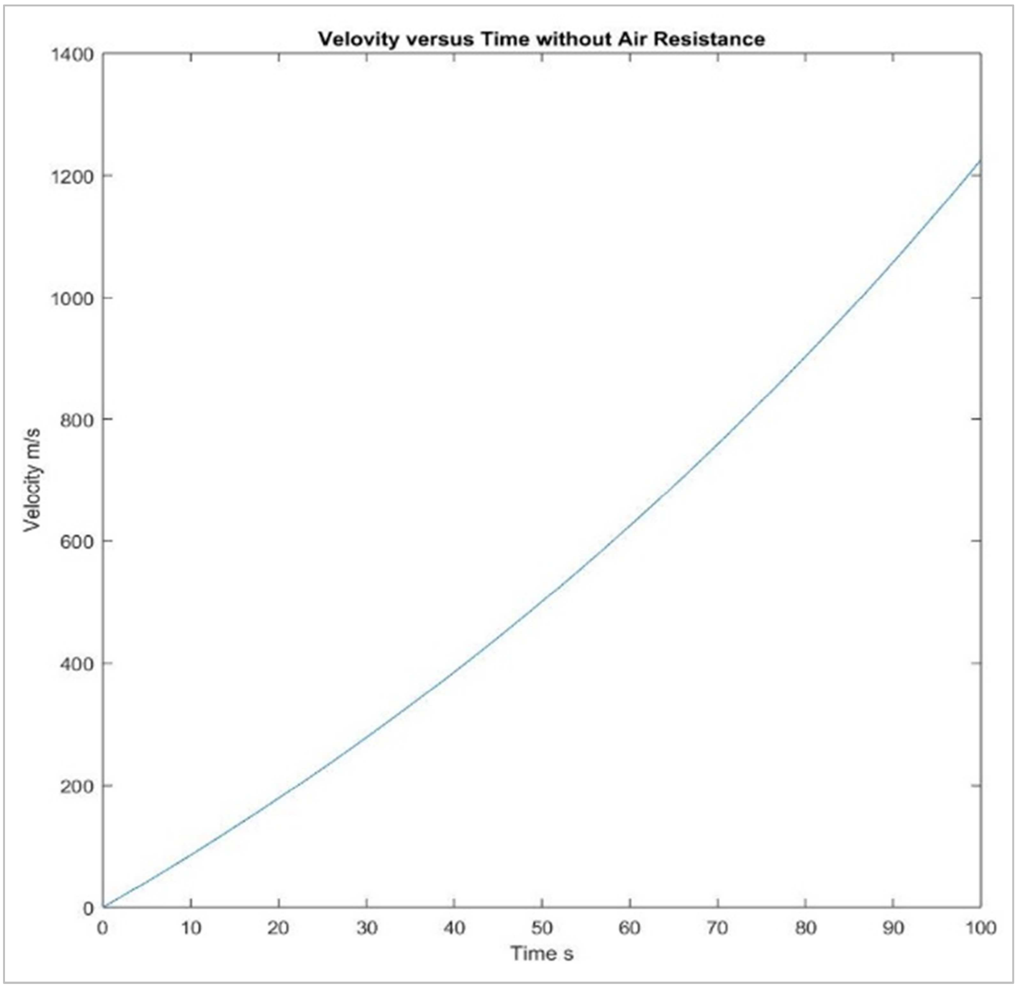

Figure 3. *The velocity of the rocket as a function of time. The air resistance is neglected in this figure.*

The final velocity of the rocket is determined by the fuel thrust (which is the main effect) and also the height dependence of the gravity constant which is determined by the formula below

$$\mathrm{F} = G\frac{m_1 m_2}{r^2}$$

Here F is the gravity between two objects. And the distance r is related to the height in our rocket model.

When the effect of air resistance is taken into consideration, however, the graph becomes more complex, see the Figure 4.

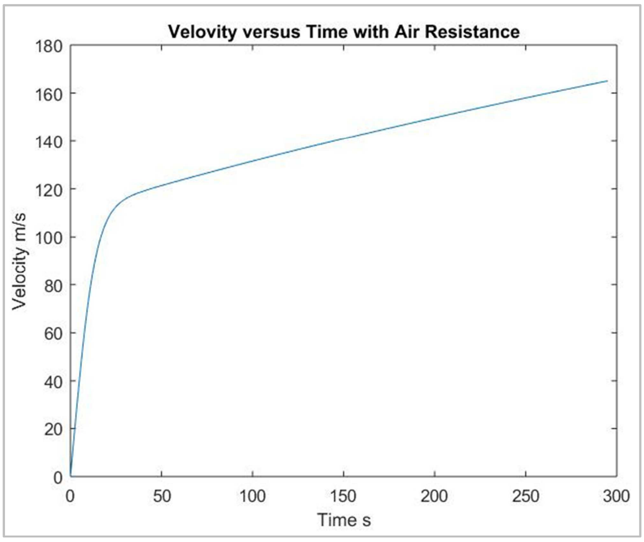

Figure 4. *The velocity of the rocket as a function of time. The air resistance is neglected in this figure.*

Despite the effect of air resistance, the rocket accelerates rapidly in the initial stage of launching, due to the small velocity of the rocket. With the time increasing, the height increases which brings smaller gravity on the rocket and contribute to the larger acceleration of the rocket. However, the velocity of the rocket increases fast and brings large air resistance. This reduces the acceleration of the rocket. And the slope in Figure 4 steadily decreases. As its velocity reaches a particular value, air resistance starts to have a telling effect on the Rocket. At T=24s the acceleration already decreases to 0.86m/s^2, a value smaller than one. Due to the high velocity, air resistance acts against the rocket throughout the whole launching process, causing the acceleration to decrease dramatically and finally to negative values, because the fuel will have been exhausted.

4. Summary

In summary, this paper proposes a theoretical model with differential-integral equation to simulate the rocket launch in the air by including the fuel thrust, gravity, and the air resistance. As the gravity depends on the height and the air resistance depends on both the height and the velocity of the rocket, it becomes much more complicated to solve the equation for the rocket motion analytically. Therefore, the formula is written in a discrete form and solved numerically. The acceleration of the rocket increases with time at the early stage of the rocket launch due to the smaller gravity with the increasing height. At the larger stage, the velocity of the rocket becomes larger, which brings large air resistance to the rocket and reduce the acceleration. In the final stage, there is no air force because the air density becomes zero, and the gravity is also negligible. The rocket is pushed only by the fuel thrust with constant acceleration. A realistic simulation for the rocket motion is obtained, and the importance of each factor on the rocket is analyzed. In future studies, it is suggested that the detailed effects of the earth rotation and the wind effects in the air should be incorporated as a part of the model.

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