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The Diffusion Behavior of the System Driven by Non-Gaussian Noise or Its Time Derivative

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Abstract: The diffusion behavior of the system driven by the non-Gaussian noise and its time derivative are investigated in detail. The temperature dependence of the noise spectral profile is firstly analyzed using Monte Carlo simulations, which is shown that the spectrum of the non-Gaussian noise is a decreasing function of temperature when the frequency is sufficient small. By contrast, its derivative is Gaussian and vanishes for the low frequency. In addition, diffusion behavior of the system subjected to non Gaussian noise or its time derivative are more detailed discussed within the framework of the generalized Langevin equation. It is particularly revealed that the system driven by the internal non-Gaussian noise behaves as normal diffusion for various temperatures, while the time derivative of the non-Gaussian noise induces ballistic diffusion of a free system and the variance is sensitive to the initial condition which implies the breaking of the ergodicity.

Keywords: Non-Gaussian Noise, Ballistic Diffusion, Monte Carlo Simulations

1. Introduction

During last decades, there are a wealth of researches on the noise-induced phenomena [1, 2, 3, 4, 5]. A large number of examples are stochastic resonance of the systems [6, 8], noise-induced transitions [9], noise-induced phase transitions [10, 11, 12, 14], noise-induced transport [14, 15], etc. In many situations, the noises actually play a significant role in inducing new ordering phenomena. Vast majority of studies on the noise-induced phenomena indicated above assume that the noise source has a Gaussian distribution (either white or colored). However, there are some experimental evidences, particularly in sensory and biological systems [16], offer strong indications that in some of these phenomena the noise source could be non-Gaussian. Examples are current measurements through voltage-sensitive ion channels in a cell membrane or experiments on the sensory system of rat skin [17, 18]. Previous studies on the role of non Gaussian noises on some noise-induced phenomena have shown the possibility of strong effects on the system's response. For instance, enhancement of the stochastic resonance in a double well potential driven by a colored non-Gaussian noise [19]. Besides, an enhancement of current due to the non-Gaussian character of the noise appears in the ratchet potential [20]. Moreover, in the anti-tumor model with correlation between multiplicative non-Gaussian noise and additive Gaussian-colored noise, it has been found that an increase in both the non-Gaussian noise intensity and the departure from the Gaussian noise can accelerate the transition from the disease state to the healthy state [21]. These results motivate the interest in analyzing the effect of non-Gaussian noises on the diffusion behavior of the system. The substantial progress that has been achieved towards an understanding of anomalous diffusive behavior of a system during the last few years show that system driven by non-Gaussian noise exhibit anomalous diffusive behavior and their stationary state are non-Gaussian, such as Lévy flight [22, 23]. Whether a non-Gaussian noise will always induce abnormal diffusion? and what is the property of the time derivative of the non-Gaussian noise? These questions are required to be addressed and investigated.

Based on the above considerations, we present a more detailed discussion of the diffusion behavior of the system subjected to the non-Gaussian noise or its time derivative. This paper is organized as follows. In Sec. 2, we introduce the non-Gaussian noise and its time derivative and present their stationary distribution and spectral density. In Sec. 3, we discuss the numerical results for the system subjected to the noise and its time derivative. Finally, we summarize the main results and give a brief conclusion in Sec. 4.

2. Non-Gaussian Noise and Its Time Derivative

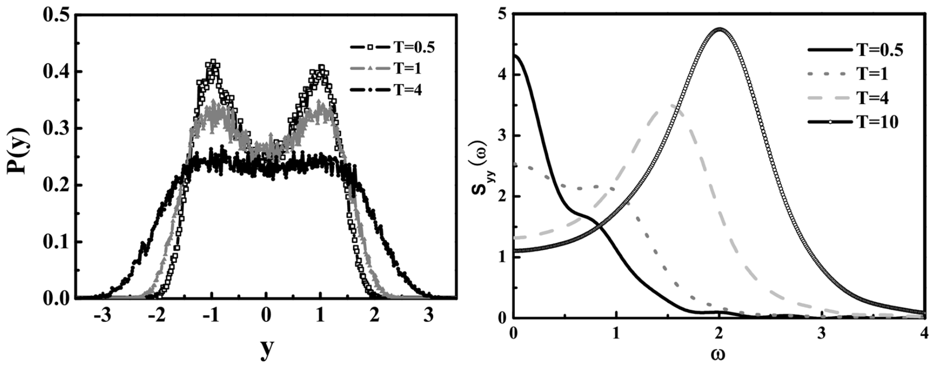
We consider the dynamics of non-Gaussian noise described by the following Langevin equation:

$$\dot{y} = z(t), \ \ \dot{z} = -\bar{\gamma} z + y - y^3 + \eta(t), \tag{1}$$

where $\eta(t)$ is the Gaussian white noise which satisfies $\langle \eta(t) \rangle = 0$ and the fluctuation-dissipation relation $\langle \eta(t)\eta(s) \rangle = 2\bar{\gamma} k_b T \delta(t-s)$, $\bar{\gamma}$ denotes the damping coefficient, k_b and T are the Boltzmann constant and temperature, respectively. We can obtain the stationary distribution and correlation function of $y(t)$ by simulating Eq. (1)numerically. The spectral density of $y(t)$ is the Fourier transform of the correlation function which is defined as

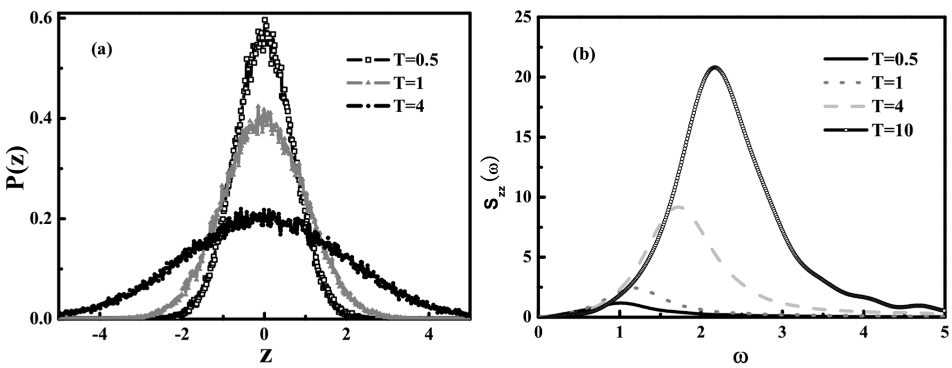
$$s(\omega, T) = 2Re \int_0^\infty \langle y(0)y(t) \rangle_T \, e^{-i\omega t} dt. \tag{2}$$

In Figure 1, we present the stationary distribution $p(y)$ and the spectral density of the $y(t)$ for various values of the temperature.



(a) (b)

Figure 1. (a) The stationary distribution $p(y)$ (b)Spectral densities of $y(t)$ for different values of temperature. The parameters used is $\bar{\gamma} = 0.4$



(a) (b)

Figure 2. (a) The stationary distribution $p(z)$ (b)Power spectrum of $z(t)$ for some value of temperature. The parameters used is $\bar{\gamma} = 0.4$.

It is obvious that the stationary distribution for different temperatures is non-Gaussian. Note that the spectral density of $y(t)$ is temperature-dependent. At low temperature $(e.g.\ T = 0.5 \ \ and \ \ T = 1)$, the spectral density has a high peak at $\omega = 0$. For a relatively high temperature$(e.g.\ T = 4 \ \ and \ \ T = 10)$, the peak around a finite frequency is generated and it shifts to large frequency with the increase of temperature. The reason for the phenomena can be explained as follows: at finite temperature, the probability for the particle that jumps from a well to the other is given by the Kramers rate $R \equiv \frac{1}{\sqrt{2\pi\gamma_1}} \exp[-\frac{1}{4\gamma_1 T}]$. For the case of low temperature, the intensity of the white noise is not large enough to make the particle jump from one well to the other and the hopping rate for the particle is small, in this case, the particle remains in one well for a long time. As a consequence, the spectrum has a high zero-frequency peak, which implies similar spectral profile to Ornstein-Uhlenbeck noise. With the increase of temperature, the hopping rate between the two well grows, then the peak at $\omega = 0$ reduces and a peak around a finite frequency is generated. In such case, the spectral profile behaves similarly as the harmonic noise. If the temperature continues to rise, the spectral profile does not change but the peak shifts to large frequency. An interesting phenomena is that when the frequency is sufficient small the spectrum decreases with the temperature. Due to decoherence rate being proportional to $S(\omega, T)$, when a two state quantum system is resonant with this spectral component, the quantum decoherence rate is a decreasing function of the temperature[24], this result can have relevant consequence for the design of quantum computers at nanometer scale.

In addition, the stationary distribution of $z(t)$ is obtained by solving the Langevin equation Eq. (1) numerically and its spectrum can be evaluated as the Fourier transform of the autocorrelation function $\langle z(0)z(t)\rangle$, the results are shown in Figure 2.

For different values of temperature, the stationary distribution $p(z)$ is Gaussian. As shown in Figure 2(b), the power spectrum of $z(t)$ possesses the feature that the low frequency part vanishes, which is similar to the spectral profile of the harmonic velocity noise.

3. Diffusion Behavior of the System Subjected to Non Gaussian Noise or Its Time Derivative

In this section, we discuss the case when the non-Gaussian noise or its time derivative is used as an internal noise to drive a system. We focus on the diffusive behavior of the particle driven by the proposed noise within the framework of the generalized Langevin equation(GLE).

The GLE including the non Gaussian noise $y(t)$ for the system in the force-free field can be written as

$$\dot{x} = v, \ \ \dot{v}(t) = -\int_0^t \gamma(t-s)v(s)ds + y(t), \tag{3}$$

where $\gamma(t)$ is the damping kernel function due to $y(t)$. The autocorrelation function of the noise satisfy the fluctuation dissipation relation$\langle y(t)y(s)\rangle = k_bT\gamma(t-s)$. In the numerically calculation, the initial condition is $x(0) = 0$, $v(0)$ obeys a Gaussian distribution with zero mean and variance v_0^2.

In Figure 3, we show the time evolution of the variance $x^2(t)$ and $v^2(t)$ that obtained by solving the GLE (3) numerically, here $k_b = 1$ is used.

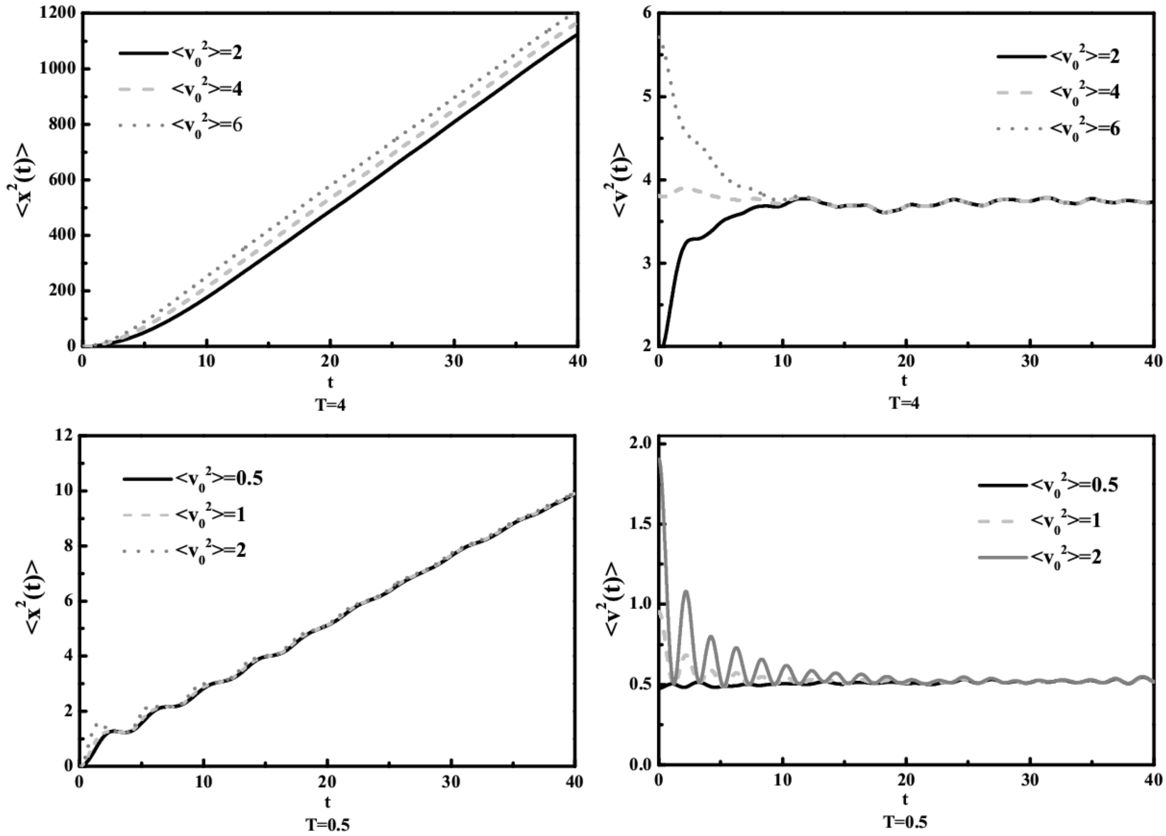


Figure 3. The time evolution of the variance $x^2(t)$ and $v^2(t)$ for the system driven by non gaussian noise $y(t)$ in the force-free field. The parameters used is $\bar{\gamma} = 0.4$.

It observed that $x^2(t)$ is proportional to t, that is to say, the system driven by the non-Gaussian noise $y(t)$ behaves as normal diffusion for various temperature. Moreover, the value of $v^2(t)$ in the long time limit does not depend on the initial distribution of the velocity and $v^2(t \to \infty) = k_b T$.

If the internal noise in GLE (3) is time derivative of y, i.e. $z(t)$, the mean-square displacement of the particle is proportional to t^2, which means the ballistic diffusion appears in the long time limit. These claims are presented in Figure 4, where we show the calculated result for the variance $x^2(t)$ and $v^2(t)$ of free particle with the initial velocity obeying a Gaussian distribution.

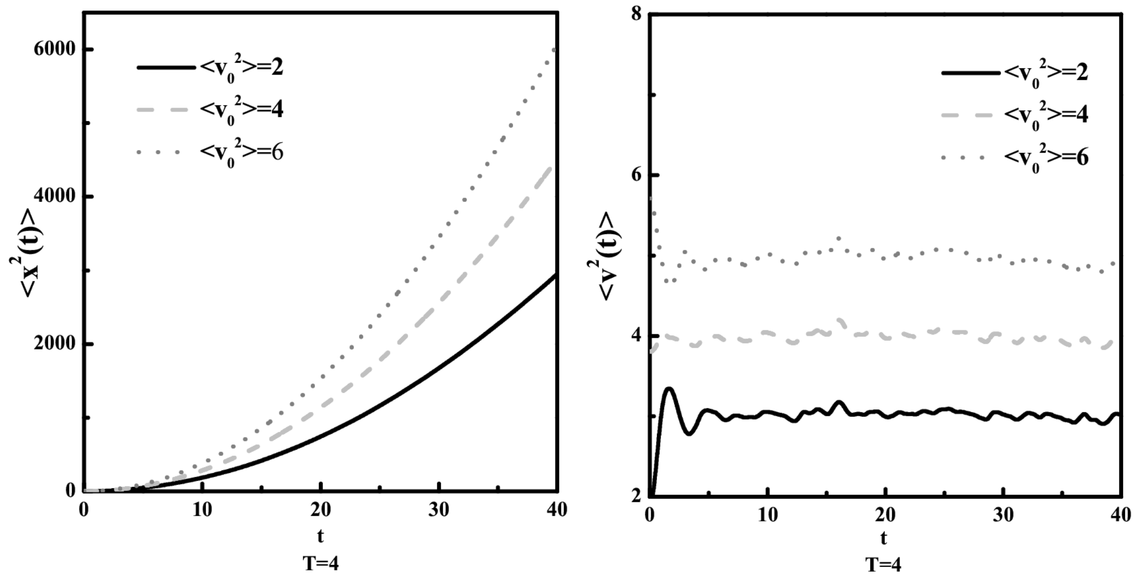


Figure 4. *The time evolution of the variance $x^2(t)$ and $v^2(t)$ for the system driven by $z(t)$ in the force-free field. The parameters used is $\bar{\gamma} = 0.4$.*

Moreover, the asymptotical result for the system subjected to $z(t)$ is sensitive to the initial condition. It is obvious that the value of $v^2(t \to \infty)$ depends on the initial distribution of the velocity, which implies the breaking of the ergodicity. Then the time derivative of the non Gaussian noise has the properties similar to the harmonic velocity noise [26, 27, 28].

4. Conclusions

In summary, the diffusion behavior of the system driven by the non Gaussian noise and its derivative have been investigated respectively. The non-Gaussian noise is produced by a system in double well potential and subjected to a white Gaussian noise. The stationary distribution and correlation function are detected by simulating Langevin equation numerically. The results have revealed that its spectrum is a decreasing function of temperature when the frequency is sufficient small, due to the temperature dependence of the spectral profile, while its derivative is Gaussian for different values of temperature. In addition, diffusion behavior of the system subjected to non Gaussian noise or its time derivative have been more detailed discussed within the framework of the generalized Langevin equation. The system in the force free field driven by the non-Gaussian noise behaves as normal diffusion for various temperatures; while the time derivative of the non Gaussian induces ballistic diffusion of a free system which leads to nonergodicity. We are confident that the present results will play part in understanding the role of non Gaussian noise-induced phenomena and stimulate further studies.

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